Name ___________________________________________________ Date____________________

# Lesson 1: Multiplying and Factoring Polynomial Expressions

## Exit Ticket

When you multiply two terms by two terms, you should get four terms. Why is the final result when you multiply two binomials sometimes only three terms? Give an example of how your final result can end up with only two terms.

Name ___________________________________________________ Date____________________

# Lesson 2: Multiplying and Factoring Polynomial Expressions

## Exit Ticket

1. Factor completely: $2a^2 + 6a + 18$

2. Factor completely: $5x^2 - 5$

3. Factor completely: $3t^3 + 18t^2 - 48t$

4. Factor completely: $4n - n^3$

Name ___________________________________________________ Date____________________

# Lesson 3: Advanced Factoring Strategies for Quadratic Expressions

## Exit Ticket

1. Use algebra to explain how you know that a rectangle with side lengths one less and one more than a square will always be 1 square unit smaller than the square.

2. What is the difference in the areas of a square and rectangle, if the rectangle has side lengths 2 less and 2 more than a square? Use algebra or a geometric model to compare the areas and justify your answer.

3. Explain why the method for factoring shown in this lesson is called the product-sum method.

Name ______________________________________________ Date____________________

# Lesson 4: Advanced Factoring Strategies for Quadratic Expressions

## Exit Ticket

1. Explain the importance of recognizing common factors when factoring complicated quadratic expressions.

2. Factor: $8x^2 + 6x + 1$

Name ___________________________________________________ Date____________________

# Lesson 5: The Zero Product Property

## Exit Ticket

1. Factor completely: $3d^2 + d - 10$

2. Solve for $d$: $3d^2 + d - 10 = 0$

3. In what ways are Problems 1 and 2 similar? In what ways are they different?

Name ___________________________________________________________ Date____________________

# Lesson 6: Solving Basic One-Variable Quadratic Equations

## Exit Ticket

1. Solve the equations:
   a. $4a^2 = 16$

   b. $3b^2 - 9 = 0$

   c. $8 - c^2 = 5$

2. Solve the equations:
   a. $(x - 2)^2 = 9$

   b. $3(x - 2)^2 = 9$

   c. $6 = 24(x + 1)^2$

Name _______________________________________________ Date____________________

# Lesson 7: Creating and Solving Quadratic Equations in One Variable

## Exit Ticket

1. The perimeter of a rectangle is $54$ cm. If the length is $2$ cm more than a number and the width is $5$ cm less than twice the same number. What is the number?

2. A plot of land for sale has a width of $x$ ft. and a length that is $8$ ft. less than its width. A farmer will only purchase the land if it measures $240$ sq. ft. What value for $x$ will cause the farmer to purchase the land?

Name_______________________________________________ Date____________________

# Lesson 8: Exploring the Symmetry in Graphs of Quadratic Functions

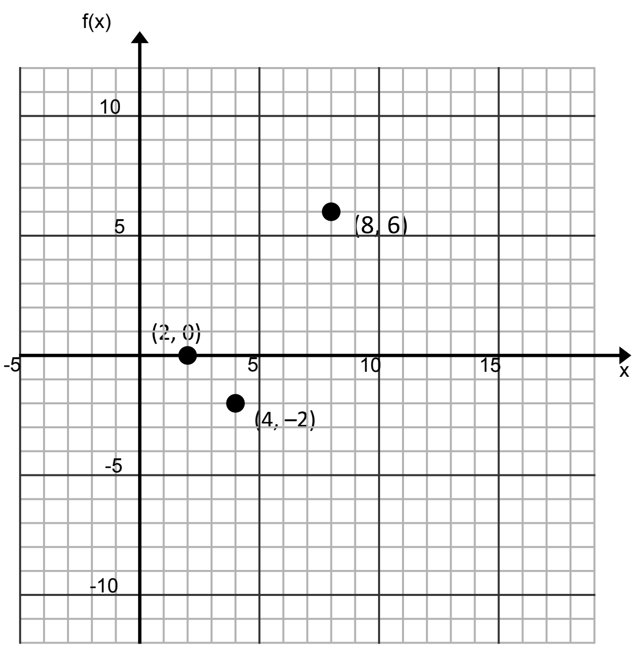
## Exit Ticket

1. If possible, find the equation for the axis of symmetry for the graph of a quadratic function with the given pair of coordinates. If not possible, explain why.

   a. $(3, 10)$ $(15, 10)$

   b. $(-2, 6)$ $(6,\ 4)$

2. The point $(4, -2)$ is the vertex of the graph of a quadratic function. The points $(8, 6)$ and $(2, 0)$ also fall on the graph of the function. Complete the graph of this quadratic function by first finding two additional points on the graph. (If needed, make a table of values on your own paper.) Then answer the questions on the right.




   a. Find the $y$-intercept.

   b. Find the $x$-intercept(s).

   c. Find the interval on which the rate of change is always positive.

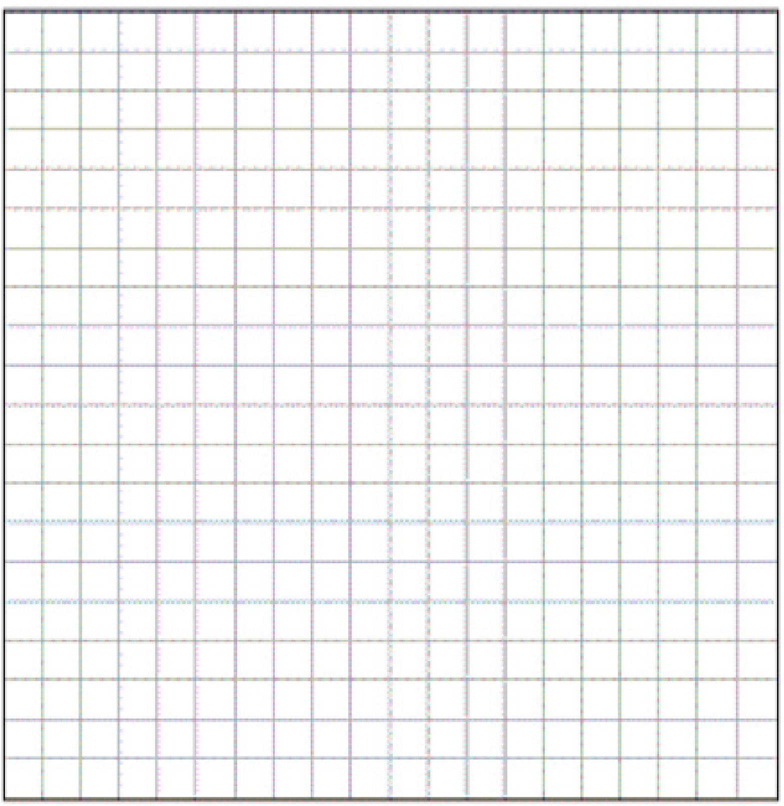
   d. What is the sign of the leading coefficient for this quadratic function? Explain how you know.

Name ______________________________________________ Date____________________

# Lesson 9: Graphing Quadratic Functions from Factored Form, $f(x) = a(x - m)(x - n)$

## Exit Ticket

Graph the following function and identify the key features of the graph: $h(x) = -3(x - 2)(x + 2)$.

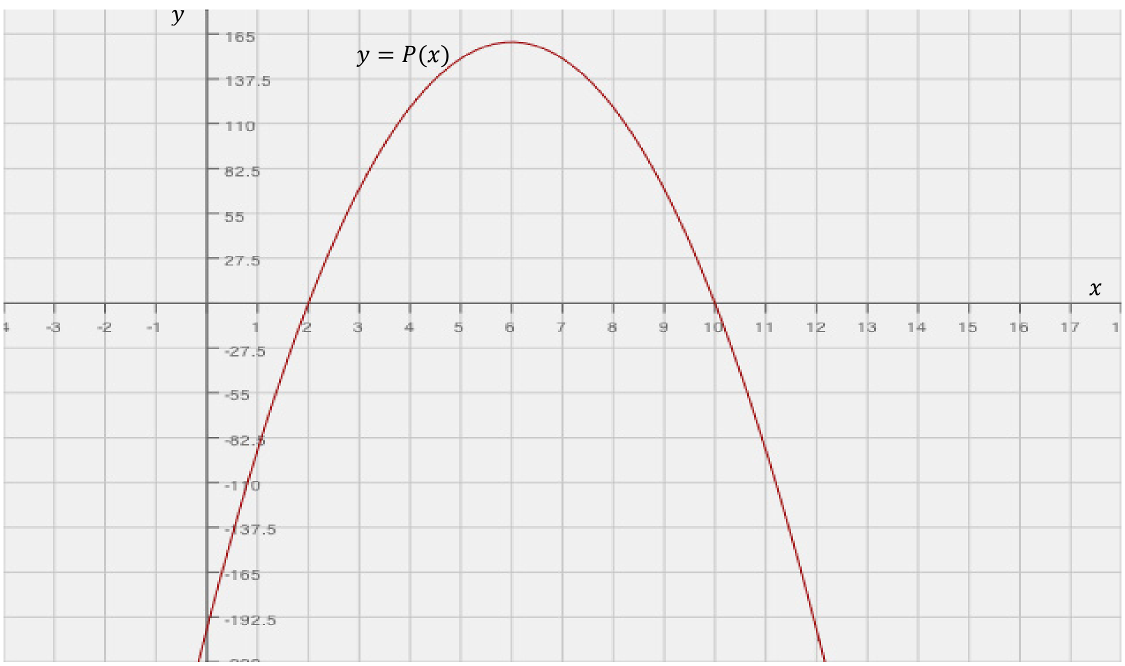


Name ______________________________________________ Date____________________

# Lesson 10: Interpreting Quadratic Functions from Graphs and Tables

## Exit Ticket

1. A toy company is manufacturing a new toy and trying to decide on a price that will result in a maximum profit. The graph below represents profit ($P$) generated by each price of a toy ($x$). Answer the questions based on the graph of the quadratic function model.




a. If the company wants to make a maximum profit, what should the price of a new toy be?

b. What is the minimum price of a toy that will produce profit for the company? Explain your answer.

c. Estimate the value of $P(0)$ and explain what the value means in the problem and how this may be possible.

d. If the company wants to make a profit of $\$137$, for how much should the toy be sold?

e. Find the domain that will only result in a profit for the company and find its corresponding range of profit.

f. Choose the interval where the profit is increasing the fastest:
$[2, 3]$, $[4, 5]$, $[5.5, 6.5]$, $[6, 7]$

g. The company owner believes that selling the toy at a higher price will result in a greater profit. Explain to the owner how selling the toy at a higher price will affect the profit.

Name ______________________________ Date ______________

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of $x$ for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.

   a. Perimeter:

   b. Area:

   c. Are both your answers polynomials? Explain why or why not for each.

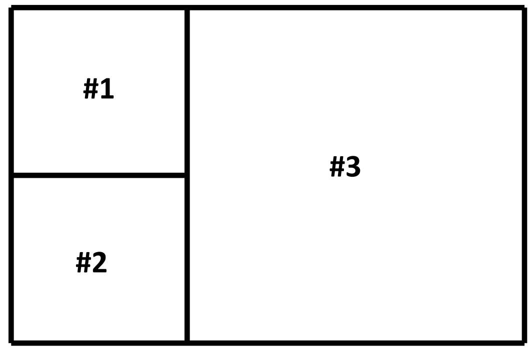
   d. Is it possible for the perimeter of the rectangle to be $16$ units? If so, what value(s) of $x$ will work? Use mathematical reasoning to explain how you know you are correct.

e. For what value(s) of the domain will the area equal zero?

f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares #1 and #2 in the figure below. All three shapes are squares. The area of square #1 equals that of square #2 and each can be represented by the expression $4x^2 - 8x + 4$.




a. Find the side length of the father's plot, which is square #3, and show or explain how you found it.

b. Find the area of the father's plot and show or explain how you found it.

c. Find the total area of all three plots by adding the three areas and verify your answer by multiplying the outside dimensions. Show your work.

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle, so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from the time it was thrown could be modeled closely by the function,

$$h(t) = -16t^2 + 64t + 80$$

where $h(t)$ represents the height of the ball in feet after $t$ seconds.

   a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

   b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

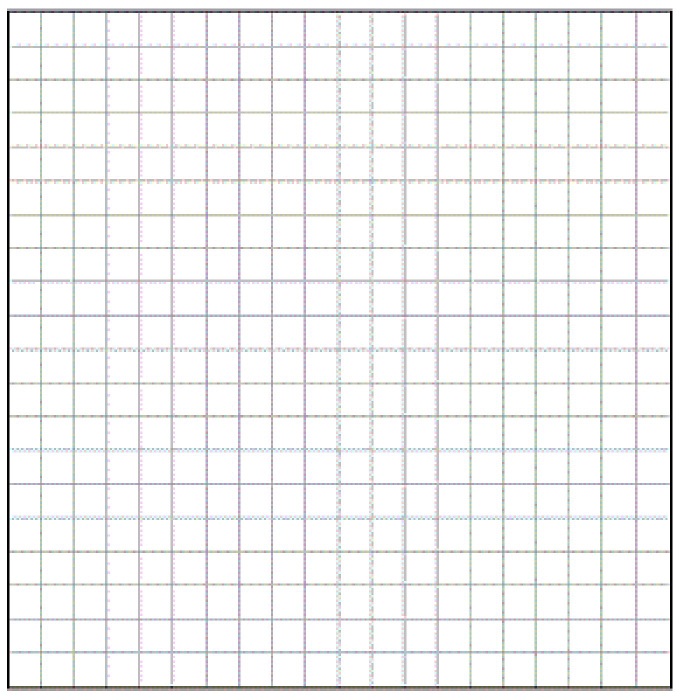
   c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

   d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

e. How long is the ball in the air? Explain your answer.

f. State the domain of the function and explain the restrictions on the domain based on the context of the problem.

g. Graph the function indicating the vertex, axis of symmetry, intercepts, and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.



h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

Name ___________________________________________________ Date____________________

# Lesson 11: Completing the Square

## Exit Ticket

Rewrite the expression $r^2 + 4r + 3$, first by factoring, and then by completing the square. Which way is easier? Explain why you think so.

Name ___________________________________________________ Date____________________

# Lesson 12: Completing the Square

## Exit Ticket

1. Complete the square: $ax^2 + x + 3$

2. Write the expression for the profit, $P$, in terms of $q$, the quantity sold, and $s$, the selling price, based on the data collected below on sales and prices. Use the examples and your notes from class to then determine the function that represents yearly profit, $P$, in terms of the sales, $s$, given the production cost per item is $30.

| Selling Price $ ($s$) | Quantity sold ($q$) |
|---|---|
| 100 | 7,000 |
| 200 | 6,000 |
| 500 | 3,000 |
| 600 | 2,000 |
| 800 | 0 |

Name ___________________________________________________ Date____________________

# Lesson 13: Solving Quadratic Equations by Completing the Square

## Exit Ticket

1. Solve the following quadratic equation both by factoring and by completing the square: $\frac{1}{4}x^2 - x = 3$

2. Which method do you prefer to solve this equation? Justify your answer using algebraic reasoning.

Name ___________________________________________________ Date____________________

# Lesson 14: Deriving the Quadratic Formula

## Exit Ticket

Solve for $R$ using any method. Show your work.

$$\frac{3}{2}R^2 - 2R = 2$$

Name ___________________________________________________ Date____________________

# Lesson 15: Using the Quadratic Formula

## Exit Ticket

1. Solve the following equation using the quadratic formula: $3x^2 + 6x + 8 = 6$

2. Is the quadratic formula the most efficient way to solve this equation? Why or why not?

3. How many zeros does the function $f(x) = 3x^2 + 6x + 2$ have? Explain how you know.

Name ______________________________________________ Date____________________

# Lesson 16: Graphing Quadratic Equations from the Vertex Form, $y = a(x - h)^2 + k$

## Exit Ticket

1. Compare the graphs of the function, $f(x) = -2(x + 3)^2 + 2$ and $g(x) = 5(x + 3)^2 + 2$. What do the graphs have in common? How are they different?

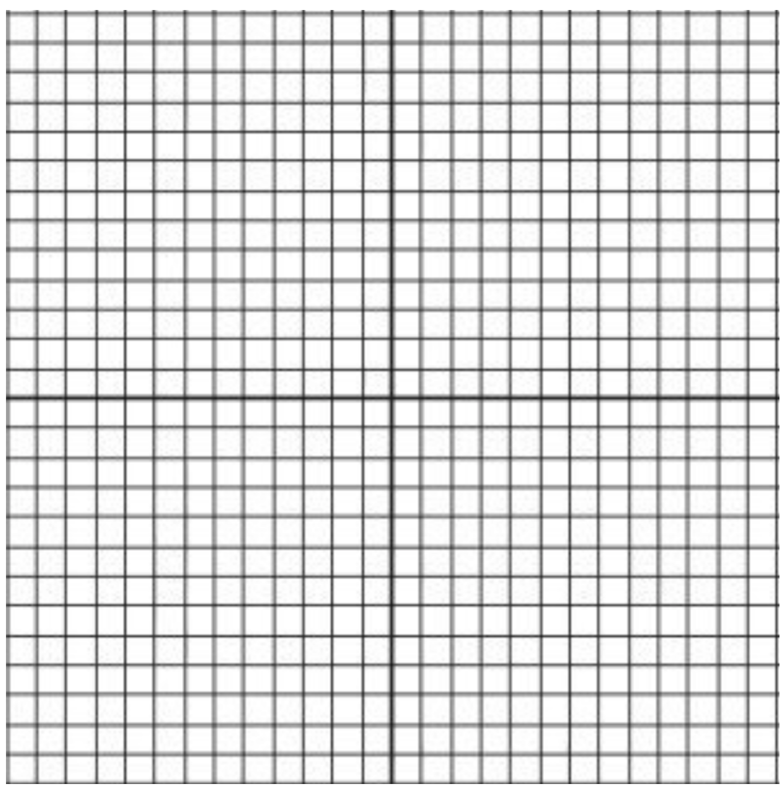
2. Write two different equations representing quadratic functions whose graphs have vertices at $(4.5, -8)$.

Name ___________________________________________________ Date____________________

# Lesson 17: Graphing Quadratic Functions from the Standard Form, $f(x) = ax^2 + bx + c$

## Exit Ticket

Graph $g(x) = x^2 + 10x - 7$ and identify the key features (e.g., vertex, axis of symmetry, $x$- and $y$-intercepts).



Name ___________________________________________________ Date____________________

# Lesson 18: Graphing Cubic, Square Root, and Cube Root Functions

## Exit Ticket

1. Describe the relationship between the graphs of $y = x^2$ and $y = \sqrt{x}$. How are they alike? How are they different?

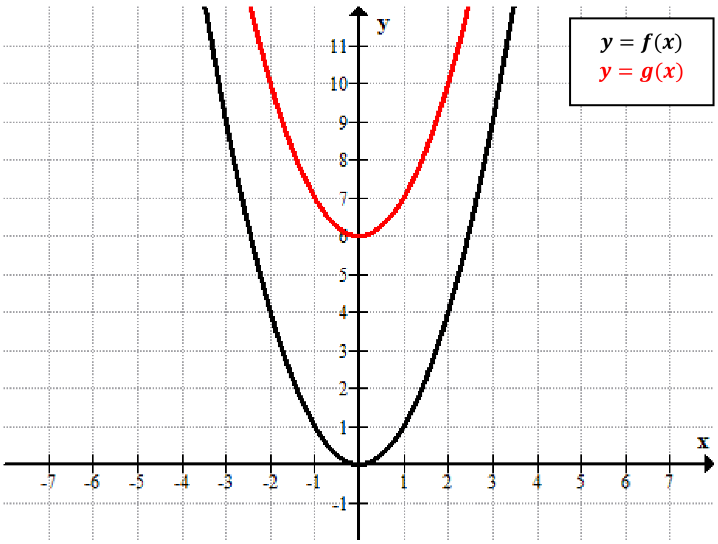
2. Describe the relationship between the graphs of $y = x^3$ and $y = \sqrt[3]{x}$. How are they alike? How are they different?

Name ______________________________ Date________________

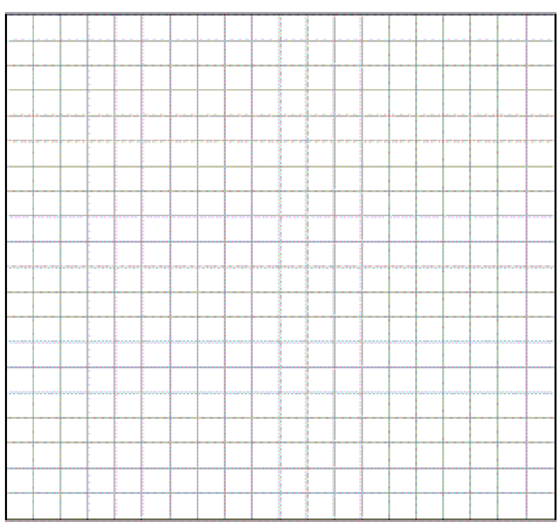
# Lesson 19: Translating Functions

## Exit Ticket

1. Ana sketched the graphs of $f(x) = x^2$ and $g(x) = x^2 - 6$ as shown below. Did she graph both of the functions correctly? Explain how you know.




2. Use transformations of the graph of $f(x) = \sqrt{x}$ to sketch the graph of $f(x) = \sqrt{x-1} + 3$.



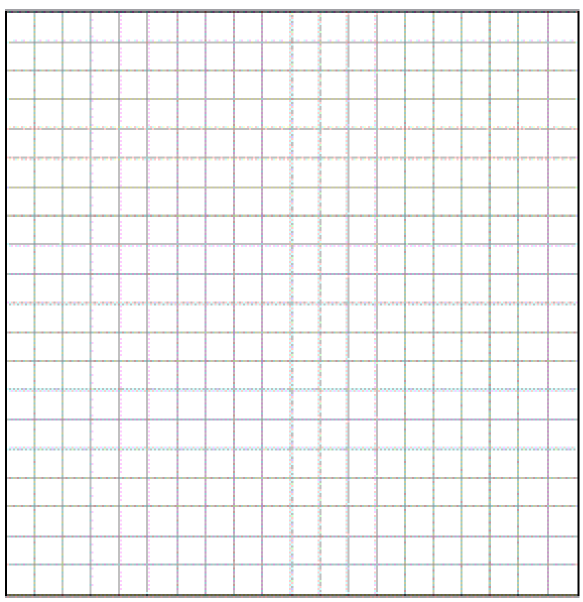
Name ___________________________________________________ Date____________________

# Lesson 20: Stretching and Shrinking Functions

## Exit Ticket

1. How would the graph of $f(x) = \sqrt{x}$ be affected if it were changed to $g(x) = -2\sqrt{x}$?

2. Sketch and label the graphs of both $f$ and $g$ on the grid below.

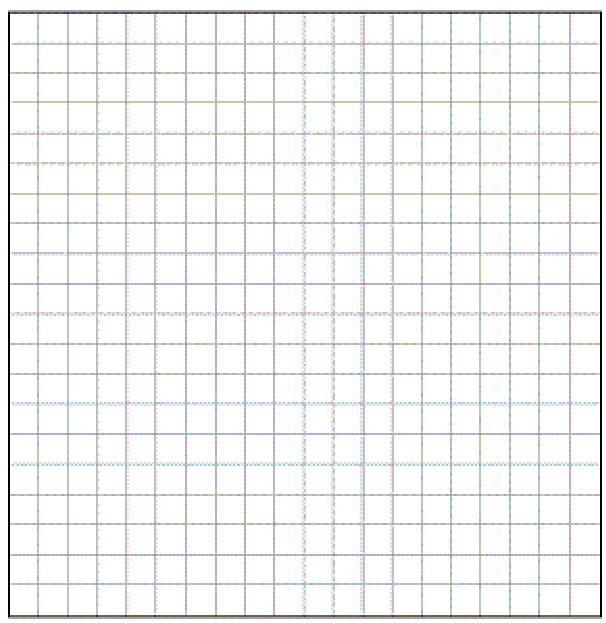


Name ______________________________________________ Date____________________

# Lesson 21: Transformations of the Quadratic Parent Function, $f(x) = x^2$

## Exit Ticket

Describe in words the transformations of the graph of the parent function $f(x) = x^2$ that would result in the graph of $g(x) = (x + 4)^2 - 5$. Graph the equation $y = g(x)$.



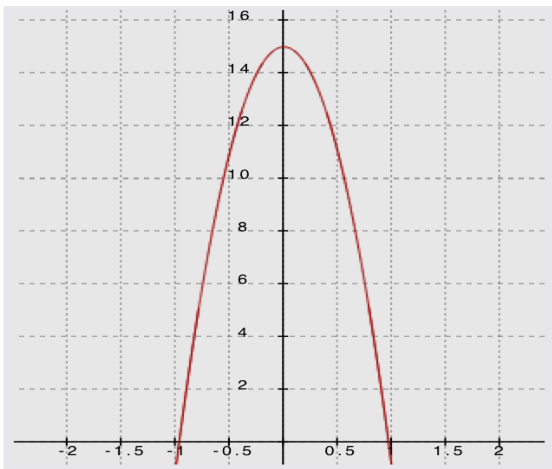
Name ___________________________________________________ Date____________________

# Lesson 22: Comparing Quadratic, Square Root, and Cube Root Functions Represented in Different Ways

## Exit Ticket

1. Two people, each in a different apartment building, have buzzers that don't work. They both must throw their apartment keys out of the window to their guests, who will then use the keys to enter.

   Tenant one throws the keys such that the height-time relationship can be modeled by the graph below.




   Tenant two throws the keys such that the relationship between the height of the keys (in feet) and the time that has passed (in seconds) can be modeled by $h(t) = -16t^2 + 18t + 9$.

   a. Whose window is higher? Explain how you know.

b. Compare the motion of tenant one's keys to that of tenant two's keys.

c. In this context, what would be a sensible domain for these functions?

Name ___________________________________________________ Date____________________

# Lesson 23: Modeling with Quadratic Functions

## Exit Ticket

What is the relevance of the vertex in physics and business applications?

Name ___________________________________________________ Date____________________

# Lesson 24: Modeling with Quadratic Functions

## Exit Ticket

Write a quadratic function from the following table of data:

| Fertilizer Impact On Corn Yields | | | | | |
|---|---|---|---|---|---|
| Fertilizer, $x$ (kg/m$^2$) | $0$ | $100$ | $200$ | $300$ | $400$ |
| Corn Yield, $y$ (1000 bushels) | $4.7$ | $8.7$ | $10.7$ | $10.7$ | $8.7$ |

## Lagrange's Interpolation Method: An Extension for Accelerated Students

Lagrange's Interpolation Method allows mathematicians to write a polynomial from a given set of points. Because three points determine a unique quadratic function, students can use interpolation to write a quadratic function without having to solve a system of equations to find the coefficients.

Given the points $(a, b)$, $(c, d)$, $(e, f)$, the quadratic function defined by these points can be written:

$$f(x) = b \cdot \frac{(x-c)(x-e)}{(a-c)(a-e)} + d \cdot \frac{(x-a)(x-e)}{(c-a)(c-e)} + f \cdot \frac{(x-a)(x-c)}{(e-a)(e-c)}$$

This works because, for each $x$ substituted into the function, two of the terms disappear by the zero–multiplication rule and the third term divides to $f(x) \cdot 1$. For example, write the quadratic function uniquely defined by the points: $(-1, 2)$ $(2, 23)$ $(-4, -1)$.

$$f(x) = 2 \cdot \frac{(x-2)(x+4)}{(-3)(3)} + 23 \cdot \frac{(x+1)(x+4)}{(3)(6)} - 1 \cdot \frac{(x+1)(x-2)}{(-3)(-6)}$$

then $f(2) = 2 \cdot \frac{(2-2)(2+4)}{(-3)(3)} + 23 \cdot \frac{(2+1)(2+4)}{(3)(6)} - 1 \cdot \frac{(2+1)(2-2)}{(-3)(-6)}$

and $f(2) = 0 + 23 \cdot \frac{(3)(6)}{(3)(6)} - 0$

so $f(2) = 23 \cdot 1 = 23$

This process can be repeated for each of the three points, and so this function is clearly a degree two polynomial containing the three given points. This form may be considered perfectly acceptable; however, multiplying out and collecting like terms, we can re-write this function in standard form:

$$\begin{aligned} f(x) &= \frac{-2}{9}(x^2 + 2x - 8) + \frac{23}{18}(x^2 + 5x + 4) - \frac{1}{18}(x^2 - x - 2) \\ 18f(x) &= -4x^2 - 8x + 32 + 23x^2 + 115x + 92 - x^2 + x + 2 \\ 18f(x) &= 18x^2 + 108x + 126 \\ f(x) &= x^2 + 6x + 7 \end{aligned}$$

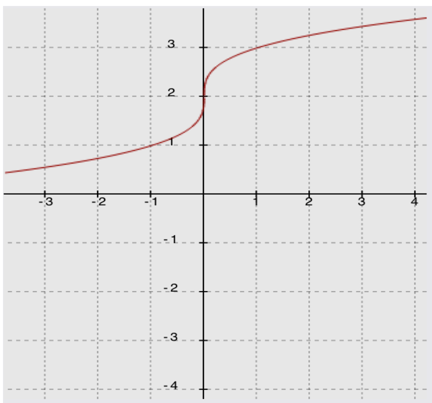
For students who love a challenge, design a short set of exercises with which accelerated students may practice interpolation. These exercises should not necessarily reduce to integer or even rational coefficients in standard form, and students may want to consider the potential pros and cons of leaving the function in its original interpolated form.

Name ________________________________________________ Date ______________________

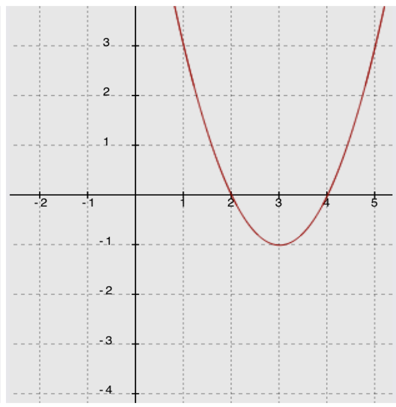
1. Label each graph with the function it represents; choose from those listed below:

$f(x) = 3\sqrt{x}$ $\qquad g(x) = \frac{1}{2}\sqrt[3]{x}$ $\qquad h(x) = -5x^2$

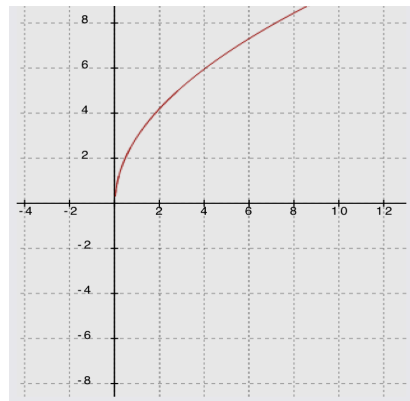
$k(x) = \sqrt{x+2} - 1$ $\qquad m(x) = \sqrt[3]{x} + 2$ $\qquad n(x) = (x-3)^2 - 1$



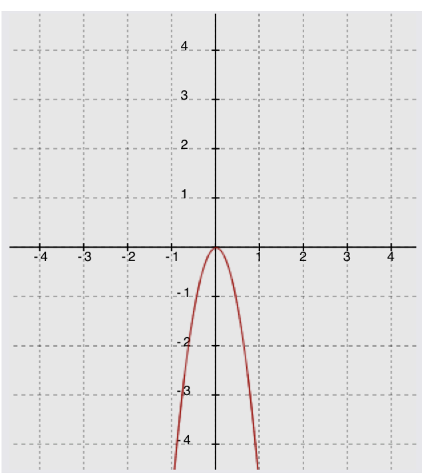
Function _______________



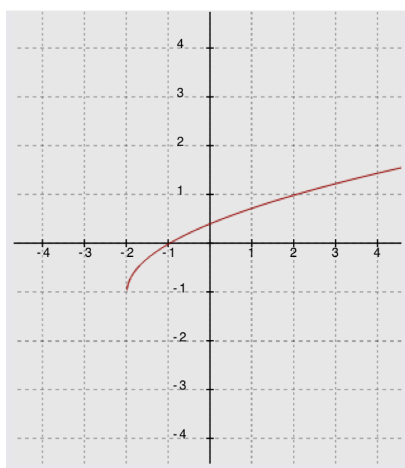
Function _______________



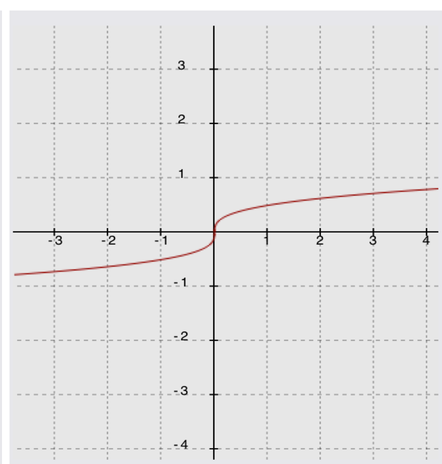
Function _______________



Function _______________



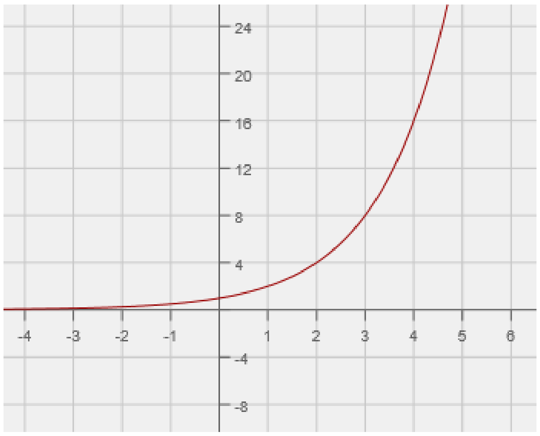
Function _______________



Function _______________

2. Compare the following three functions:

i. A function $f$ is represented by the graph below:




ii. A function $g$ is represented by the following equation:

$$g(x) = (x-6)^2 - 36$$

iii. A linear function $h$ is represented by the following table:

| $x$ | $-1$ | $1$ | $3$ | $5$ | $7$ |
|---|---|---|---|---|---|
| $h(x)$ | $10$ | $14$ | $18$ | $22$ | $26$ |

For each of the following, evaluate the three expressions given and identify which expression has the largest value and which has the smallest value. Show your work.

a. $f(0),\ g(0),\ h(0)$

b. $\frac{f(4)-f(2)}{4-2},\ \frac{g(4)-g(2)}{4-2},\ \frac{h(4)-h(2)}{4-2}$

c. $f(1000)$, $g(1000)$, $h(1000)$

3. An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, $t$, and the height of the arrow in meters, $h$, is given by:

$$h(t) = -4.9t^2 + 29.4t + 2.5$$

a. Complete the square for this function. Show all work.

b. What is the maximum height of the arrow? Explain how you know.

c. How long does it take the arrow to reach its maximum height? Explain how you know.

d. What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds? Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds and explain the difference in the context of the problem.

e. How long does it take the arrow to hit the ground? Show your work or explain your answer.

f. What does the constant term in the original equation tell you about the arrow's flight?

g. What do the coefficients on the second- and first-degree terms in the original equation tell you about the arrow's flight?

4. Rewrite each expression below in expanded (standard) form:

a. $\left(x+\sqrt{3}\right)^2$

b. $\left(x-2\sqrt{5}\right)\left(x-3\sqrt{5}\right)$

c. Explain why, in these two examples, the coefficients of the linear terms are irrational and why the constants are rational.

Factor each expression below by treating it as the difference of squares:

d. $q^2-8$

e. $t-16$

5. Solve the following equations for $r$. Show your method and work. If no solution is possible, explain how you know.

   a. $r^2 + 12r + 18 = 7$ b. $r^2 + 2r - 3 = 4$ c. $r^2 + 18r + 73 = -9$

6. Consider the equation: $x^2 - 2x - 6 = y + 2x + 15$ and the function: $f(x) = 4x^2 - 16x - 84$ in the following questions:

   a. Show that the graph of the equation, $x^2 - 2x - 6 = y + 2x + 15$, has $x$-intercepts at $x = -3$ and 7.

b. Show that the zeros of the function, $f(x) = 4x^2 - 16x - 84$, are the same as the $x$-values of the $x$-intercepts for the graph of the equation in part (a). (i.e., $x = -3$ and 7)

c. Explain how this function is different from the equation in part (a)?

d. Identify the vertex of the graphs of each by rewriting the equation and function in the completed-square form, $a(x-h)^2 + k$. Show your work. What is the same about the two vertices? How are they different? Explain why there is a difference.

e. Write a new quadratic function, with the same zeros, but with a maximum rather than a minimum. Sketch a graph of your function, indicating the scale on the axes and the key features of the graph.